FDTD CALCULATION OF COUPLING COEFFICIENT BETWEEN TWO RESONATORS

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ABSTRACT

A coupling coefficient is characterized in the time domain instead of the frequency domain commonly employed. The FDTD method is well suited to its time domain calculation, resulting in a fast and simple procedure. Various techniques unique to time-domain calculation is explained with practical examples.

INTRODUCTION

A coupling constant between two resonators is one of the most important parameters for designing a band-pass filter. Though it has usually been calculated by the even and odd mode frequency of a coupled resonator [1][2], it is analytically obtained only for simple structures. Thus, the FDTD method, which is extensively used for the electromagnetic wave analysis, could be applied to find it. The procedure to obtain the coupling coefficient looks straightforward ; one assumes a system of coupled resonators with I/O ports and excites the input port by a Gaussian pulse. The output response is Fourier-transformed and two resonant frequencies are found to put in

$$k = \frac{2|f_o - f_e|}{f_o + f_e} \quad (1)$$

But we should make use of the advantage of time domain calculation as long as we rely on the FDTD method. The coupling constant can be defined in the time domain and by using it the cpu time substantially decreases compared with the conventional FDTD analysis. The principle and some examples are presented to elucidate the physics of resonator coupling together with the way to use it.

PRINCIPLE AND METHOD OF CALCULATION

Coupling between resonators or propagating waves is often compared to coupled pendulums. The energy in one oscillating system is transferred to the other system through the coupling mechanism. It arises in the time domain and space domain for resonators and propagating waves, respectively. Thus, the coupling coefficient is defined by the ratio of the period for the self-oscillation and energy transfer [3] as

$$k = 2\frac{T_0}{T_m} \quad (2)$$

First, we take an example shown in Fig.1 and calculate the coupling coefficient by a conventional FDTD procedure. Mur's 1st order absorbing boundary is introduced for each terminal. A Gaussian pulse

$$H_x = \exp[-(\Delta t \cdot n - 3T)^2 / T^2] \quad (3)$$

is applied at a terminal, where Δt=4.81 pS and T=12.49 pS. The response E_y at the other terminal is drawn in Fig.2 (a) and its Fourier transform is in (b), respectively. The obtained coupling coefficient is shown in Table.1 as well as the cpu time.

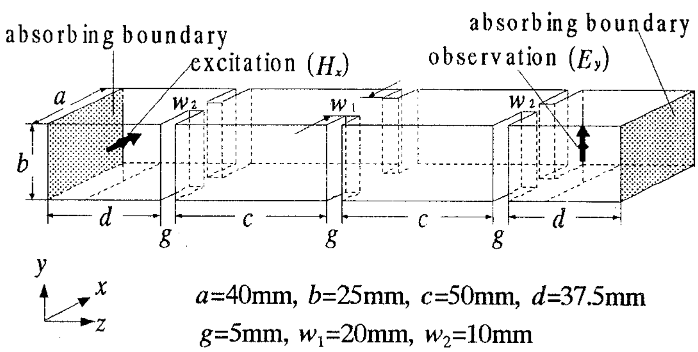

Fig.1 Configuration for calculating the coupling coefficient of two TE_{101} modes via the FDTD method in the frequency domain.

Table 1 Calculated value of coupling coefficient and cpu time for frequency and time domain FDTD analysis.

	k	Cpu time (sec)
Freq. domain	0.0314	163.5 (50000 steps) + 17.2 (FFT)
Time domain	0.0322	25.8 (10000 steps)

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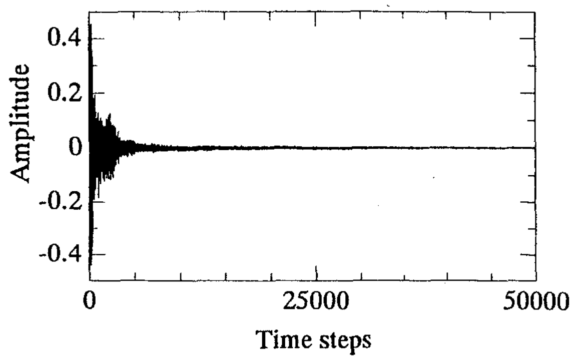
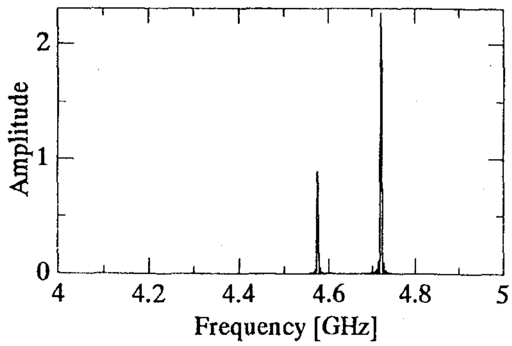


(a) Time domain



(b) Frequency domain

Fig.2 The output response for a Gaussian pulse excitaion.

Now, the time domain calculation uses a Gaussian modulated continuous wave as an excitation.

$$H_x = \sin(2\pi f_0 \Delta t \cdot n)\exp[-(\Delta t \cdot n - 3T)^2 / T^2] \qquad (4)$$

Therefore we need a rough idea for the resonant frequency of the resonator, which is discussed later. Figure 3 shows the analyzed structure which is of the same dimension as Fig.1 except for the external W/Gs. Time domain response in Fig.4 (a)(b) clearly expresses the energy exchange between two resonators, though they are the variation of electric field E_y, more precisely. We can calculate the coupling coefficient from the period in Fig.4 and that in Fig.5 which shows a part of Fig.4 (b) in an expanded time scale. In other words, we do not need an integration to obtain the total electromagnetic energy in each resonator, but need only the one component of the electric field at any place in a resonator. The calculation result is compared with that of the frequency domain in Table 1. The cpu time would decrease further if one notices the response is already stable at 3000 th step in Fig.4.

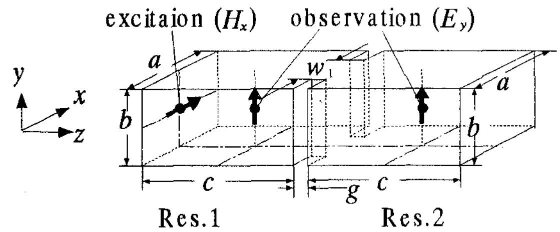
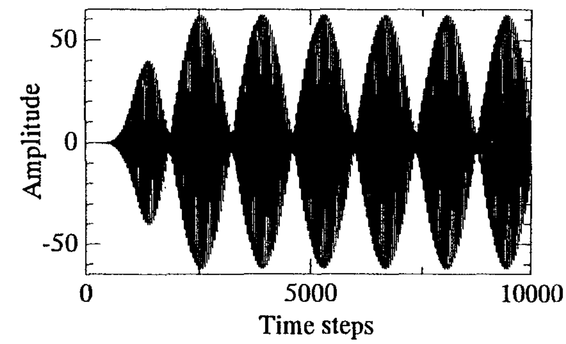
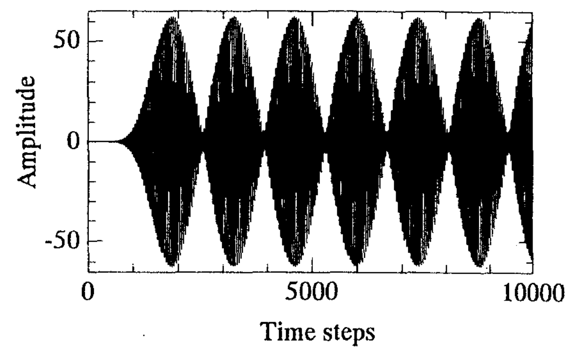


Fig.3 Configuration for calculating the coupling coefficient in the time domain.



(a) Resonator 1



(b) Resonator 2

Fig.4 The response of E_y at the center of each resonator in Fig.3 for a Gaussian modulated continuous wave .

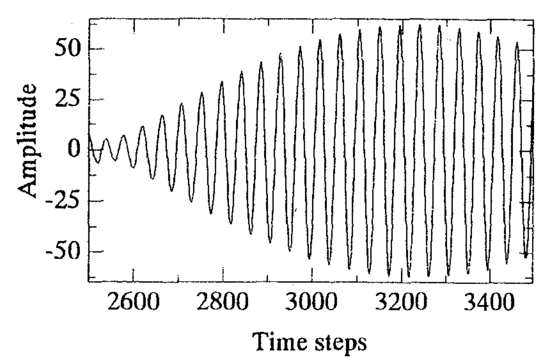


Fig.5 The expanded wave form of Fig 4 (b) along the time axis.

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SOME OTHER EXAMPLES AND DISCUSSIONS

1) Different resonators of same resonant frequency

The coupled resonator system in Fig.6 is composed of two same resonators but they are coupled at the side wall 90° rotated. Therefore the equivalent LC circuit (if obtained) looked inside from the coupling port should be different each other. Hence, this system is considered as the titled case. The computed result (Fig.7) indicates that the energy exchange is incomplete though the energy in Res.2 exchanges completely. The present property is also analytically obtained by two coupled different LC circuits of the same resonant frequency. The coupling coefficient was 0.0423 for Fig.6.

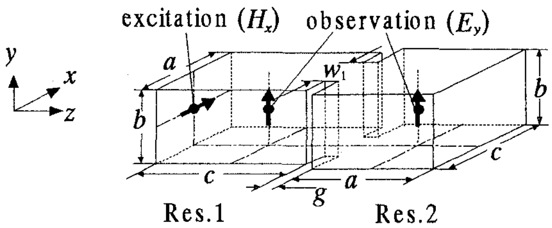
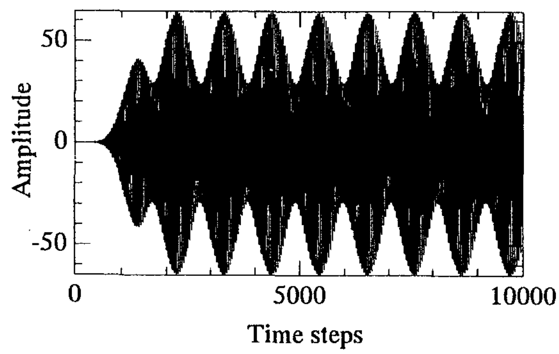
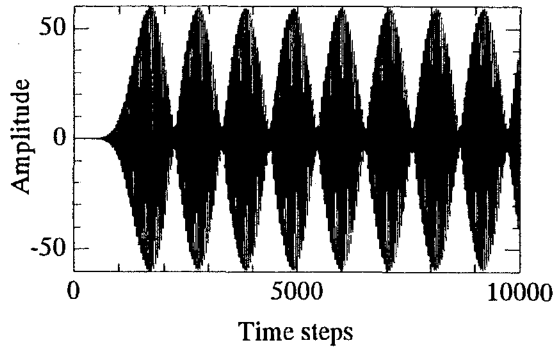


Fig.6 Coupling of two different resonators of the same resonant frequency.



(a) Resonator 1



(b) Resonator 2

Fig.7 The response of E_y at the center of each resonator in Fig.6.

2) The case when center frequency of excitation is shifted

The center frequency f_0 in eq.(4) is set exactly at the average of the even and odd mode frequency for Fig.3. However, we do not know the exact value but know a roughly estimated value. Thus, one should evaluate the error from the frequency shift. Figure 8 shows the response at the center of Res.2 in Fig.3 when f_0 is 4.80 GHz, being 133 MHz higher than that for Fig.4. Although the energy exchange is incomplete again, the calculated k value was 0.0319, being only 1% lower than that in Table 1.

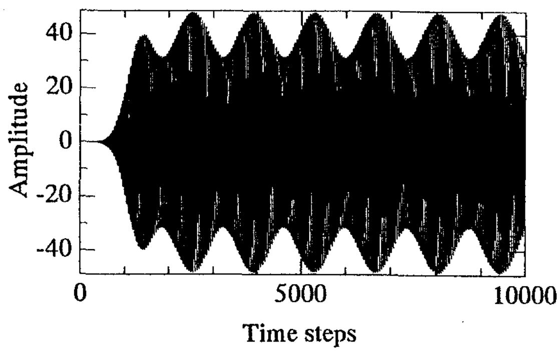


Fig.8 The response of E_y at the center of Resonator 1 when the excitation frequency is shifted upwards.

3) Coupling of degenerate modes

Two degenerate modes in a resonator couple each other by a proper perturbation. A typical example is shown in Fig.9 where TE_{101} and TE_{011} modes couple by a square corner cut.[2] Figure 10 indicates that complete energy exchange takes place between two modes. The calculated k value was 0.0183 for this case.

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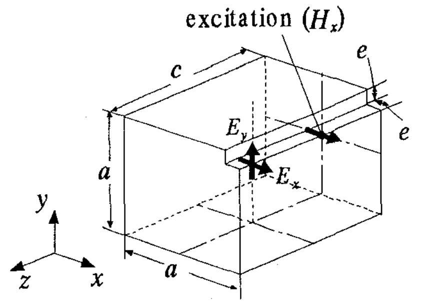
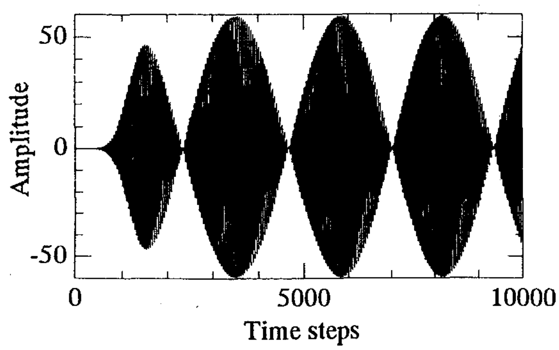
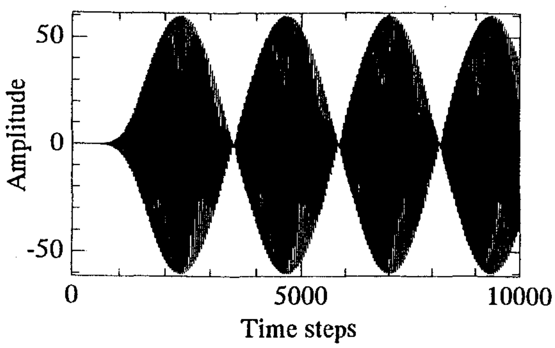


a=40mm, c=50mm, e = 5mm

Fig.9 Coupling of two degenerate TE_{101} and TE_{011} modes in a rectangular waveguide resonator of square cross section.



(a) E_y (TE_{101} mode)



(b) E_x (TE_{011} mode)

Fig.10 The response at the center of the resonator in Fig.9.

4) Coupling of microstrip resonators

Microstrip resonators shown in Fig.11 is a good example for the planar structures. An experiment has been carried out with the external lines as indicated in the figure, resulting in such a loose coupling that $|S_{21}|$ is less then -30dB. The parameters for FDTD analysis are f_0=1.3 GHz, Δt=0.589 pS, T=0.375 nS and excitation is made by a current source close to the open end of one resonator, deleting the external lines. Both results are in good agreement as shown in Fig.12, which verifies the FDTD analysis.

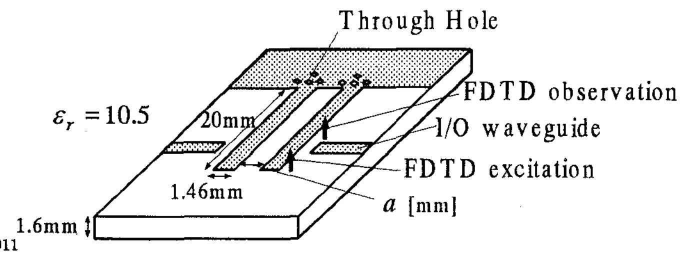


Fig.11 Configuration of microstrip resonators.

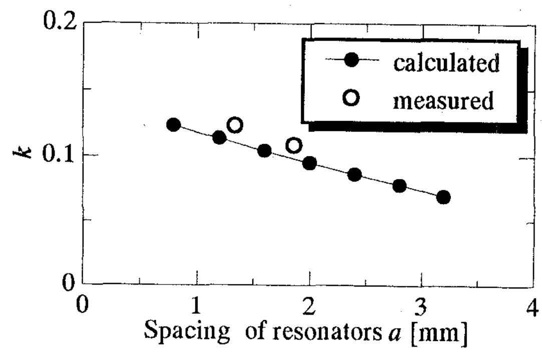


Fig.12 Change of coupling coefficient with respect to the spacing of resonators.

CONCLUSION

Coupling coefficient of two resonators are obtained numerically by the FDTD method. Taking the most of the FDTD calculation, the definition in the time domain is utilized, resulting in a simple and fast estimation of coupling coefficients.

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[3] W. H. Louisell ," Coupled Mode and Parametric Electronics" John Wiley & Sons, USA, New York, London, pp. 3-15 (1960)